

# Astrophysical Implications of the Induced Neutrino Magnetic Moment from Large Extra Dimensions

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## Abstract

Theories involving extra dimensions, a low ( $\sim \text{TeV}$ ) string scale and bulk singlet neutrinos will produce an effective neutrino magnetic moment which may be large ( $\lesssim 10^{-11} \mu_B$ ). The effective magnetic moment increases with neutrino energy, and therefore high energy reactions are most useful for limiting the allowed number of extra dimensions. We examine constraints from both neutrino-electron scattering and also astrophysical environments. We find that supernova energy loss considerations require a number of extra dimensions,  $n \geq 2$ , for an electron neutrino-bulk neutrino Yukawa coupling of order 1.

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The study of string theory has yielded many beautiful theoretical results. Although direct confrontation with experiments is still elusive, the effort has nonetheless given us many new ways of looking at the physical world. Weak coupling string theory predicts [1]

$$M_S = M_P \sqrt{k\alpha_{GUT}}, \quad (1)$$

where  $\alpha_{GUT}$  is the unified gauge coupling constant and  $k$  is an integer of order one denoting the level of the Kac-Moody algebra. In this case the string scale,  $M_S$  is close to the Plank scale,  $M_P$ . Recently, Witten [2] observed that in nonperturbative string theory,  $M_S$  could be lower. Lykken [3] and Antoniadis [4] discussed the possibility that the string scale could be close to the weak scale, which has many interesting phenomenological consequences. If the string scale where gravity becomes strong is around the TeV scale, one needs to understand why the observed Newton's constant at large distances is small. Here one elicits the help of extra dimensions inherent in string theories. It is well known that string theory can be consistently formulated in 10 or 11 dimensions. Hence 6 or 7 extra spatial dimensions exist which must be compactified. The weakness of gravity is then due its spreading in these  $n$  extra dimensions [5]. The relation that replaces Eq.(1) is given by

$$M_P^2 = M_*^{n+2} V_n \quad (2)$$

where  $V_n$  is the volume of the extra space and  $M_*$  is the higher dimensional Planck scale. The two scales  $M_S$  and  $M_*$  are related but the exact relation is not needed in this investigation. For toroidal compactification we have

$$V_n = (2\pi)^n R_1 R_2 \dots R_n \quad (3)$$

and  $R_i (i = 1, 2, \dots n)$  are the radii of the extra dimensions. For simplicity usually one sets all the radii to be equal and denoted by  $R$ , which is referred to as the symmetric compactification case. These theories predict a deviation from the Newtonian  $1/r$  law of gravitational potential at small distances. The fact that this law is well tested above a distance of 1 mm leads to the conclusion that  $n \geq 2$  for  $M_* = 1$  TeV for the symmetric case. Current proposed experiments may test smaller distance scales [6]. Recently [7] proposed an alternative to the usual compactification of extra dimensions. Here, the important effect of the extra dimension is to alter the four dimensional metric term. In this scenario the extra dimensions are not compactified and gravitational experiments are not necessarily a test of the fundamental scaling law of Eq. 2.

Independently of the exact nature of the scaling law to which gravity is sensitive, theories which involve extra dimensions can also provide a mechanism for producing a small neutrino mass. In the following, we briefly review this mechanism and then point out that it will also generate a large neutrino magnetic moment. This can be used to put constraints on the number of extra dimensions involved in producing the neutrino mass.

It is well known that the Standard Model (SM) is phenomenally successful in describing experiments. To accommodate this in the above described extra dimensional world one localizes the SM fields on a hypersurface with three spatial dimensions or 3-brane [8]. Due to the conservation of gauge flux all charged states under the SM group are expected to be trapped on the 3-brane [9]. Such a scenario with the SM residing on a 3-brane and gravity propagating in the bulk of a 10 or 11-dimensional world has thus far not been ruled out by experiments for  $n \geq 2$  and  $M_* \geq \text{O}(10 \text{ TeV})$ . To date the most stringent constraint on this scenario comes from analysis of supernova energy loss due to Kaluza-Klein graviton emission and sets  $M_* \geq 30 \sim 50 \text{ TeV}$  [10] for  $n = 2$  symmetric compactification[11]. While the SM fields are localized on the 3-brane, there is no argument to suggest that SM singlet fields need to do the same. Thus, any gauge singlet field is allowed to be bulk matter just like the graviton.

Theories with extra dimensions provide an elegant new mechanism to generate naturally small neutrino masses,  $m_\nu \leq 10^{-3} \text{ eV}$ . Such small neutrino masses are indicated by recent experiments in solar and atmospheric neutrinos [12]. The key here is to postulate the existence of a right-handed neutrino,  $\nu_R$ , which is a bulk fermion and use it to generate a small neutrino mass [13, 14, 15], by coupling to the SM left-handed neutrino field  $\nu_L$  via the standard Higgs mechanism with a crucial suppression factor. This arises because in general the couplings between states on the brane and bulk states are suppressed by a volume factor of  $M_*^{-\frac{n}{2}} V_n^{-\frac{1}{2}}$ . If combined with the scaling law of Eq. 2, the neutrino-Higgs interaction is given by

$$\frac{y}{\sqrt{M_*^n V_n}} H \bar{\nu}_L \nu_R = \frac{y M_*}{M_P} H \bar{\nu}_L \nu_R, \quad (4)$$

where  $H$  denotes the Higgs doublet,  $y$  is the Yukawa coupling and  $\nu_R$  is taken to be the bulk fermion. Spontaneous electroweak symmetry breaking then generates a Dirac mass with the active neutrino mass given by

$$m_D = \frac{yv}{\sqrt{M_*^n V_n}} \quad (5)$$

where  $v = 247 \text{ GeV}$ . If we assume that  $y = 1$ , the scaling relation of Eq. 2 and  $M_* \sim 10 \text{ TeV}$  then  $m_D \sim 10^{-4} \text{ eV}$ .

The state  $\nu_R$  is a bulk state and hence is a linear combination of a tower of Kaluza-Klein (KK) states. In the following, we will assume that  $\nu_R$  is a Dirac particle. In this case, there is a tower of left handed states  $\nu_{kL}$ . We first illustrate the relationship of the Kaluza-Klein towers to the active neutrinos  $\nu_e$  with the case of one dimension, compactified on a circle. To further simplify the model we assume that the higher dimensional bare Dirac mass term vanishes. This can be implemented naturally under  $\mathcal{Z}_2$  orbifold compactification

[for details of this scheme see [14]]. The quantization of internal momenta in the extra dimension generates Dirac mass terms (for details see [13, 14])

$$\sum_{k=-\infty}^{\infty} m_k \bar{\nu}_{kR} \nu_{kL} + h.c., \quad m_k = \frac{k}{R} \quad (6)$$

with a mass splitting of  $1/R$ . The mixing of the bulk states with the active  $\nu_{eL}$  gives rise to universal Dirac mass terms

$$m_D \sum_{k=-\infty}^{\infty} \bar{\nu}_{kR} \nu_{eL} + h.c. \quad (7)$$

With regard to the electroweak interactions  $\nu_{kL}$  and  $\nu_{kR}$  are sterile states. Expanding Eqs. 6 and 7 gives the mass terms as:

$$m_D \bar{\nu}_{0R} \nu_{eL} + m_D \sum_{k=1}^{\infty} (\bar{\nu}_{kR} + \bar{\nu}_{-kR}) \nu_{eL} + \sum_{k=1}^{\infty} \frac{k}{R} (\bar{\nu}_{kR} \nu_{kL} - \bar{\nu}_{-kR} \nu_{-kL}) + h.c. \quad (8)$$

It is useful to define the following orthogonal states

$$\nu'_{kR} = \frac{1}{\sqrt{2}}(\nu_{kR} + \nu_{-kR}) \quad \nu''_{kR} = \frac{1}{\sqrt{2}}(\nu_{kR} - \nu_{-kR}) \quad (9)$$

and

$$\nu'_{kL} = \frac{1}{\sqrt{2}}(\nu_{kL} - \nu_{-kL}) \quad \nu''_{kL} = \frac{1}{\sqrt{2}}(\nu_{kL} + \nu_{-kL}), \quad (10)$$

which can be used to rewrite Eq.8 as

$$m_D \bar{\nu}_{0R} \nu_{eL} + \sqrt{2} m_D \sum_{k=1}^{\infty} \bar{\nu}'_{kR} \nu_{eL} + \sum_{k=1}^{\infty} \frac{k}{R} (\bar{\nu}'_{kR} \nu'_{kL} + \bar{\nu}''_{kR} \nu''_{kL}) + h.c. \quad (11)$$

The states with double prime superscripts have no low energy interactions and will be ignored in our discussions. The mass term can now be written in the familiar form of  $\bar{\nu}_L M \nu_R$  where  $\nu_L = (\nu_{eL}, \nu'_{1L}, \nu'_{2L}, \dots)$  and  $\nu_R = (\nu'_{0R}, \nu'_{1R}, \nu'_{2R}, \dots)$ . The mass matrix  $M$  for  $k+1$  states look as follows

$$M = \begin{bmatrix} m_D & \sqrt{2}m_D & \sqrt{2}m_D & \dots & \sqrt{2}m_D \\ 0 & \frac{1}{R} & 0 & \dots & 0 \\ 0 & 0 & \frac{2}{R} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & \frac{k}{R} \end{bmatrix}. \quad (12)$$

Following [16] the diagonalizing of the mass matrix leads to the mixing of the  $k$ th state to the lowest mass eigenstate  $\nu_{0L}^m$  given by

$$\tan 2\theta_k \approx \frac{2\xi}{k - \xi^2}, \quad (13)$$

for small  $\xi$  where  $\xi \equiv \sqrt{2}m_DR$ . For values of  $m_D \sim 10^{-4}\text{eV}$  and  $R \sim 1\text{mm}$  we find  $\xi \sim 10^{-1}$ . Thus the mixing of the KK states with the lowest mass eigenstate  $\nu_{0L}^m$  becomes progressively smaller as the value of  $k$  increases. A solution of the solar neutrino problem in terms of matter enhanced flavor transformation of  $\nu_{eL}$  into these bulk modes has been explored in [16]. It is also studied in the supernova collapse phase in [17]. An alternate model of neutrino mass and right handed neutrinos is provided in [18].

In this paper we study an important role that these bulk neutrino modes can play in stellar energy loss. There are several processes, such as the magnetic moment interaction, spin flip via the ordinary weak interaction and Higgs boson exchange which can populate the bulk neutrino modes, allowing energy to escape more quickly from stellar environments than in the case without neutrino mass. In the bulk neutrino scenario each mechanism is enhanced albeit differently by the volume of the compactified space. This volume enhancement is due to the large number of KK states into which the active neutrino can scatter. However, in Higgs exchange scattering, the large enhancement factor is insufficient to overcome the small Higgs-fermion couplings. In the following we focus on the transition magnetic moment interaction for the active neutrinos and the bulk modes. There will be a similar enhancement in the case of neutrino spin flip scattering, but this process is more difficult to study. In general, the rate of these processes is determined not only by the volume of the extra dimensions, but also by the energy available for the scattering.

The same enhancement factor appears in the consideration of stellar energy loss due to KK graviton states. This calculation involves theoretical modeling of KK graviton pion and nucleon couplings. Model dependence in the case of bulk neutrinos enters through the coupling of the active neutrinos to the bulk modes and the choice of Dirac or Majorana neutrinos. Here for simplicity we assume only  $\nu_{eL}$  couples to the bulk modes, ignoring the couplings of the other two flavors. We have also taken the KK neutrinos to be Dirac neutrinos. The limits we obtain below are complementary to the constraints from the graviton case although as we will show, numerically they are similar.

We begin by discussing the neutrino magnetic moment,  $\mu_{(k)}$  generated by the presence of the  $k^{th}$  bulk neutrinos. This is given by the familiar magnetic moment interaction term,  $\mu_{(k)}\bar{\psi}_{\nu_{kR}^m}\sigma_{\lambda\rho}\psi_{\nu_{0L}^m}F^{\lambda\rho}$ , written here in terms of mass eigenstates. The transition magnetic moment is calculated to be

$$\mu_{(k)} = \frac{3eG_F}{\sqrt{2}(4\pi)^2}\theta_k\left(\frac{k}{R}\right) \quad (14)$$

$$= 1.6 \times 10^{-19}\mu_B\left(\frac{m_D}{1\text{eV}}\right) \quad (15)$$

where we have used Eq. 13 for small  $\xi$ . It can be seen from this equation that the magnetic moment is independent of  $k$  and only depends on the value of  $m_D$ . The same result can

be obtained by using the mass insertion technique and can therefore be generalized to more than one dimension.

Electron neutrinos can transform to  $k$ th bulk neutrinos by way of the magnetic moment interaction at a rate proportional to  $N\alpha\mu_{(k)}^2$  where  $N$  is the number of available bulk neutrino modes, and  $\alpha$  is the fine structure constant. Transitions can take place as long as the mass of the mode is less than the available energy,  $E$ . In the case of an electron neutrino scattering into a bulk mode, the available energy is that of the incoming electron neutrino. In the case of plasmon decay, the available energy is that of the plasmon. The total effective magnetic moment,  $\mu_{eff}$  is a sum over the full multiplicity of the available KK modes. Neglecting the influence of the mass of the neutrinos in the phase space factor,

$$\mu_{eff} \approx N^{1/2}\mu_{(k)} \approx 4 \times 10^{-8} \mu_B y \left( \frac{10^{-5}}{2\pi} \right)^{n/2} A_n^{1/2} \left( \frac{E}{10 \text{ MeV}} \right)^{n/2} \left( \frac{1 \text{ TeV}}{M_*} \right)^{n/2}. \quad (16)$$

The neutrino magnetic moment depends on both the energy of the neutrinos, the number of extra dimensions and on the higher dimensional scale  $M_*$ . In equation 16,  $A_n$  is the volume of the positive hemisphere of a  $n$ -dimensional unit sphere;  $A_n = 1$  for one extra dimension. It can be seen from the above equations that a larger  $M_*$  or a larger number of extra dimensions will make the effective neutrino magnetic moment smaller. A larger available energy, however, will make the moment bigger, since there are many more energetically allowed states for the bulk neutrinos.

This effective magnetic moment expressed as Eq. 16 *does not* depend on the manner in which the volume  $V_n$  is distributed among the extra dimensions. It is also independent of the scaling relation Eq. 2. It is a consequence of the form of the Yukawa coupling given in Eq. 4. If this coupling had a different dependence on volume, then the effective magnetic moment could well depend on the scales of the extra dimensions. In contrast, limits on the higher dimensional scale from graviton emission are directly linked to Eq. 2.

There are several existing limits on the the size of the neutrino magnetic moment coming from both terrestrial experiments and astrophysical considerations. A review of the astrophysical limits may be found in [19].

One such limit may be obtained from analysis of electron recoil spectra from neutrino-electron scattering done with reactor antineutrinos. Reactor antineutrinos can have energies up to  $\sim 10$  MeV but the peak in the spectrum comes at about  $\sim 1$  MeV. At present the best limit on the neutrino magnetic moment from this method is  $\mu_\nu = 1.9 \times 10^{-10} \mu_B$  at 95% confidence [20]. Using equation 16 with  $E = 1$  MeV, and a coupling of  $y = 1$ , we see that for  $n = 1$  with a  $M_* = 1$  TeV,  $\mu_{eff} \approx 2 \times 10^{-11} \mu_B$ . Such a scenario is not ruled out by reactor neutrino experiments.

Similarly, a limit on the neutrino magnetic moment can be derived from the shape of the electron recoil spectrum from neutrino - electron scattering by solar neutrinos as measured at SuperKamiokande [21]. The neutrinos which contribute to this scattering rate are about

5 – 15 MeV and the limit on the neutrino magnetic moment is  $\mu_\nu \leq 1.6 \times 10^{-10} \mu_B$  [22]. Since these neutrinos are about an order of magnitude more energetic than the reactor neutrinos, for  $M_* = 1 \text{ TeV}$ ,  $y = 1$  and  $n = 1$ ,  $\mu_{eff} \approx 5 \times 10^{-11} \mu_B$ . Therefore, these parameters are allowed by this measurement as well.

The effective neutrino magnetic moment for available energies of 10 MeV is shown in Figure 1. This figure plots the neutrino magnetic moment against the scale  $M_*$ . Effective magnetic moments are shown for different numbers of extra dimensions. Also shown as a horizontal line is the limit from the solar neutrino data.

An astrophysical limit on the effective magnetic moment comes from plasmon decay [23] in horizontal branch stars. Plasmon decay through the magnetic moment causes an increased energy loss in these stars and decreases their lifetime. The limit on the neutrino magnetic moment from this phenomenon is given by  $\mu_\nu \leq 10^{-11} \mu_B$  [24]. The temperature of the core of these stars is on the order of 10 keV. Therefore around 10 keV is available for the neutrinos. Using Eq 16 with  $E = 0.001$ , we see that the bulk neutrino effective magnetic moment contribution for plasmon decay will be considerably smaller than that for reactor and solar neutrinos.

On the other hand, supernova neutrinos could have energies up to 30 MeV in the core of the proto-neutron star. If the neutrino magnetic moment is large, then electron neutrinos can transform into right handed bulk neutrino states and escape from the core of the neutron star. This releases energy from the proto neutron star too quickly for the neutrino signature to be in agreement with the neutrino signal from supernova 1987a. The constraint on the neutrino magnetic moment derived in this fashion is  $\mu_\nu \leq 10^{-12} \mu_B$  [25]. This limit is shown as a horizontal dashed line on Figure 1.

The dependence of available energy on the effective magnetic moment is shown in Figure 2. The limits from supernova and HB stars are shown as crosses. The calculated effective magnetic moment is plotted as solid lines for the cases of one, two and three dimensions. From Figures 1 and 2 it is clear that a single extra dimension, with a coupling of  $y = 1$  can not be reconciled with the supernova neutrino signal without further modification of the neutrino interaction and/or supernova picture. Also apparent is the dramatic effect of available energy on the effective magnetic moment. Measurements of or constraints on  $\mu_\nu$  at high energy are more successful at probing this extra dimensional model. We show explicitly the effect of  $y \neq 1$  in Figure 3.

In conclusion we have shown that if  $\nu_R$  is a bulk neutrino it can induce an effective transition magnetic moment for the active neutrino which depends on the number of extra dimensions. Neutrino magnetic moment considerations are a probe of the coupling of the bulk neutrinos to the active neutrinos. The constraints discussed here on extra dimensions can be considered independently of the gravitational limits, since limits stemming from the neutrino magnetic moment do not depend on the gravitational scaling relation. Bulk neutrino flavor transformation scenarios either in the sun or in supernovae require at least two extra dimensions, a small Yukawa coupling  $y \lesssim 10^{-1}$  or a large higher dimensional

scale  $M_*$  in order to avoid significantly altering the picture of supernova neutrino energy loss.

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Figure 1: Shows the effective neutrino magnetic moment in units on  $\mu_B$ , for the parameters  $y = 1$ ,  $E = 10 \text{ MeV}$ . The effective magnetic moment is plotted against  $M_*$  for  $n = 1, 2$  and  $3$  dimensions. Also shown as dashed lines are the limits on the moment from supernova neutrinos (lower line) and neutrino electron scattering at SuperKamiokande (upper line).

Figure 2: Shows the effective neutrino magnetic moment in units on  $\mu_B$ , for the parameters  $y = 1$ ,  $M_* = 1 \text{ TeV}$ . The effective magnetic moment is plotted against available energy for  $n = 1, 2$  and  $3$  dimensions. Also shown as crosses are the limit the moment from supernova neutrinos (lower cross) and lifetime of HB stars (upper cross).

Figure 3: Shows the effective neutrino magnetic moment in units on  $\mu_B$ , for the parameters  $E = 10 \text{ MeV}$ ,  $M_* = 1 \text{ TeV}$ . The effective magnetic moment is plotted against Yukawa coupling for  $n = 1$  and  $2$  dimensions. Also shown as horizontal lines are the limits on the moment from supernova neutrinos (lower line) and neutrino electron scattering at SuperKamiokande (upper line).





